

PROCEDURE FOR MEASURING TURBULENT VELOCITY  
PULSATIONS IN NONISOTHERMAL STREAMS

A. L. Genkin, V. I. Kukes,  
and L. P. Yarin

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The optimum relations are found between the temperatures of the thermoanemometer filament and the gas assuring the minimum error in measuring the pulsation characteristics of a flow.

As a rule, in the thermoanemometric determination of the pulsation characteristics of turbulent nonisothermal streams one uses the "method of three superheats," which allows one to isolate from the total signal of the thermoanemometer probe the components connected only with pulsations in the velocity or temperature [1]. In accordance with this method measurements are made at three different values (arbitrary in principle) of the temperature  $T_w$  of the thermoanemometer filament for the determination of  $u'$  and  $T'_g$ . From the solution of the system of heat-balance equations for the filament corresponding to measurements at different values of  $\langle \Delta T \rangle = \langle T_g - T_w \rangle$  one can find the temperature and velocity pulsations and the correlation  $\langle u' T'_g \rangle$ . In this case the accuracy of the determination of the pulsation characteristics depends on the values of  $\langle \Delta T \rangle$ . At some values of the filament superheats relative to the surrounding medium realized in experiments the measurement error proves to be unacceptably large. In this connection the choice of the optimum modes of thermoanemometer operation assuring the minimum measurement error is important. An analysis of the error in measurements of velocity pulsations in nonisothermal gas streams by a constant-temperature thermoanemometer is carried out below in application to the procedure developed in [2].

The intensity of the velocity pulsations is determined by the following relation [2]:

$$\frac{\sqrt{\langle u'^2 \rangle}}{\langle u \rangle} = \sqrt{\frac{M}{N}} = \sqrt{\Pi}, \quad (1)$$

where

$$M = k_{u_1} k_{T_1} (\langle e_2^2 \rangle k_{T_2}^2 - \langle e_3^2 \rangle k_{T_2}^2) + k_{u_2} k_{T_2} (\langle e_3^2 \rangle k_{T_1}^2 - \langle e_1^2 \rangle k_{T_1}^2) + k_{u_3} k_{T_3} (\langle e_1^2 \rangle k_{T_2}^2 - \langle e_2^2 \rangle k_{T_2}^2),$$

$$N = k_{u_1} k_{T_1} (k_{u_2}^2 k_{T_2}^2 - k_{u_3}^2 k_{T_2}^2) + k_{u_2} k_{T_2} (k_{u_3}^2 k_{T_1}^2 - k_{u_1}^2 k_{T_1}^2) + k_{u_3} k_{T_3} (k_{u_1}^2 k_{T_2}^2 - k_{u_2}^2 k_{T_2}^2),$$

$$\langle e_i^2 \rangle = \frac{\langle V_i'^2 \rangle}{\langle V_i \rangle^2} = k_{u_i}^2 \frac{\langle u'^2 \rangle}{\langle u \rangle^2} + 2k_{u_i} k_{T_i} \frac{\langle u' T'_g \rangle}{\langle u \rangle \langle T_g - T_\infty \rangle} + k_{T_i}^2 \frac{\langle T_g'^2 \rangle}{\langle T_g - T_\infty \rangle^2},$$

$$k_{u_i} = \frac{n}{2} \frac{B_1 \langle u \rangle^n}{A_1 + B_1 \langle u \rangle^n}, k_{T_i} = - \frac{\langle T_g - T_\infty \rangle}{\langle T_{w_i} - T_g \rangle},$$

$i = 1, 2, 3$ .

We designate the relative rms errors of the quantities  $\sqrt{\langle e_i^2 \rangle}$ ,  $k_{u_i}$ , and  $k_{T_i}$  ( $i = 1, 2, 3$ ) as  $\epsilon_e$ ,  $\epsilon_{k_u}$ , and  $\epsilon_{k_T}$ , respectively, and we assume that they are independent. In this case the relative rms error  $\epsilon_u$  in the measurement of velocity pulsations can be calculated in the following way [3]:

$$\epsilon_u = \frac{\Delta(1 \sqrt{\langle u'^2 \rangle} / \langle u \rangle)}{1 \sqrt{\langle u'^2 \rangle} / \langle u \rangle} = \quad (2)$$

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$$= \frac{1}{2\Pi} \sqrt{\sum_{i=1}^3 \left[ \left( \frac{\partial \Pi}{\partial \sqrt{\langle e_i^2 \rangle}} \right)^2 \langle e_i^2 \rangle \varepsilon_e^2 + \left( \frac{\partial \Pi}{\partial k_{u_i}} \right)^2 k_{u_i}^2 \varepsilon_{k_u}^2 + \left( \frac{\partial \Pi}{\partial k_{T_i}} \right)^2 k_{T_i}^2 \varepsilon_{k_T}^2 \right]}. \quad (2)$$

Using Eq. (1), we obtain

$$\begin{aligned} \sum_{i=1}^3 \left( \frac{\partial \Pi}{\partial \sqrt{\langle e_i^2 \rangle}} \right) \langle e_i^2 \rangle &= [a_2^2 a_3^2 (a_3 - a_2)^2 (a_1 m R - 1)^2 + \\ &+ a_1^2 a_3^2 (a_3 - a_1)^2 (a m^2 R - 1)^2 + a_1^2 a_2^2 (a_1 - a_2) \times \\ &\times (a_3 m R - 1)^2] / [a_1 (a_3^2 - a_2^2) + a_2 (a_1^2 - a_3^2) + a_3 (a_2^2 - a_1^2)]^2, \\ \sum_{i=1}^3 \left( \frac{\partial \Pi}{\partial k_{u_i}} \right)^2 k_{u_i}^2 &= \{m^2 a_1^2 a_2^2 a_3^2 [(a_1 - a_3)^2 (a_2 m + R)^2 + \\ &+ (a_2 - a_1)^2 (a_3 m + R)^2 + (a_3 - a_2)^2 (a_1 m + R)^2] / [a_1 (a_3^2 - \\ &- a_2^2) + a_2 (a_1^2 - a_3^2) + a_3 (a_2^2 - a_1^2)]^2, \\ \sum_{i=1}^3 \left( \frac{\partial \Pi}{\partial k_{T_i}} \right)^2 k_{T_i}^2 &= [d_1^2 a_2^2 a_3^2 (a_2 - a_3)^2 + d_2^2 a_1^2 a_3^2 (a_3 - a_1)^2 + \\ &+ d_3^2 a_1^2 a_2^2 (a_1 - a_2)^2] / [a_1 (a_3^2 - a_2^2) + a_2 (a_1^2 - a_3^2) + a_3 (a_2^2 - a_1^2)]^2. \end{aligned} \quad (3)$$

From Eq. (3) it is seen that the error  $\varepsilon_U$  depends on the relative errors  $\varepsilon_{k_U}$ ,  $\varepsilon_{k_T}$ , and  $\varepsilon_e$ , the ratios of the coefficients of sensitivity  $a_i$ , the ratio  $m$  of the temperature and velocity pulsations, and the correlation coefficient  $R$ .

The error  $\varepsilon_{k_U}$  can be calculated from the equation

$$\varepsilon_{k_u} = \frac{1}{k_u} \frac{\partial k_u}{\partial \langle u \rangle} \Delta \langle u \rangle. \quad (4)$$

An analysis of test data on the dependences of the output voltage of a thermoanemometer on the velocity shows that the quantity  $1/k_U \cdot \partial k_U / \partial u$  varies within the limits of  $2 \cdot 10^{-2} - 10^{-3}$  sec/m upon variation of the velocity from 5 to 30 m/sec, respectively. Since the relative error in the measurement of the average velocity by a Pitot tube in a turbulent stream is 0.1, we find that  $\varepsilon_{k_U} < 5 \cdot 10^{-2}$ . On the basis of analogous estimates one can assume that  $\varepsilon_{k_T} < 5 \cdot 10^{-2}$ . As for  $\varepsilon_e$ , it is determined by the class of accuracy of the instruments measuring the output voltage of the thermoanemometer. When using apparatus of the DISA Co. the error  $\varepsilon_e$  does not exceed 0.01. Some results of a calculation of the error by Eqs. (2)-(3) with variation of the quantities  $a_i$ ,  $m$ , and  $R$  in the limits of  $10 \leq a_1 \leq 100$ ,  $0.5 \leq a_2 \leq 10$ ,  $0.5 \leq a_3 \leq 3$ ,  $0.1 \leq m \leq 1$ , and  $-1 \leq R \leq 1$  are presented in Fig. 1A and B.

The calculations show that the error  $\varepsilon_U$  decreases with an increase in the maximum superheat of the filament and a decrease in the minimum superheat. In this case the maximum superheat has the greatest influence on  $\varepsilon_U$  (Fig. 1A). As for the intermediate and minimum superheats, their variation within certain (rather broad) limits has a weak effect on the measurement error.

On this basis, when measuring the intensity of turbulent velocity pulsations with an error not exceeding 10-20% one must vary the temperature  $T_W$  of the thermoanemometer filament in the course of the experiment so that the condition  $a_3 \leq 1-1.5$  is satisfied at the maximum superheat, the condition  $3 < a_2 < 7$  is satisfied at the intermediate superheat, and the condition  $a_1 \geq 15-20$  is satisfied at the minimum superheat.

It should be noted that the maximum superheat of the filament is limited above and depends on the filament material. For the platinum-rhodium detector of type 55A75 of the DISA company, e.g., the maximum filament temperature must not exceed  $T_{W \max} = 1100^\circ\text{K}$ , and the indicated level of measurement accuracy can be reached with  $\langle T_g \rangle \leq (500-600^\circ\text{K})$ .

As for the minimum superheat, it is limited in value and depends on the sensitivity and stability of the thermoanemometric apparatus. The condition  $a_1 \approx 15$  is satisfied, e.g., if  $\langle T_g \rangle \sim 500^\circ\text{K}$  and  $T_W \sim 550^\circ\text{K}$ . At a filament superheat of  $50^\circ$  a modern thermoanemometric apparatus (such as an apparatus of DISA) operates stably. It is desirable, however, that measurements corresponding to the minimum superheat be conducted in the mode a resistance thermometer [4]. In this mode, as calibrations show, the condition  $a_1 \geq 30-50$  is always satisfied.

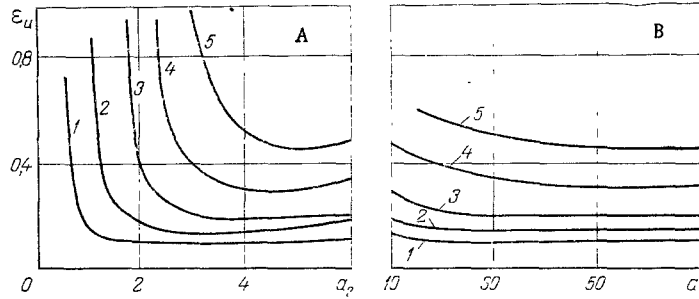


Fig. 1. Dependence of  $\epsilon_u$  on ratios of coefficients of sensitivity  $a_2$  and  $a_1$  at different values of  $a_3$ : a) dependence  $\epsilon_u(a_2)$  at  $a_1 = 50$ ; B)  $\epsilon_u(a_1)$  at  $a_2 = 5$ ;  $m = 0.5$ ;  $R = 0$ ; 1)  $a_3 = 0.5$ ; 2) 1; 3) 1.5; 4) 2; 5) 2.5.

The theoretical calculations of errors presented above agree well with the results of a statistical analysis of the experimental data described in [5].

#### NOTATION

$T$	is the temperature;
$\langle u \rangle, u'$	are the mean pulsation and components of velocity, respectively;
$\langle V \rangle, V'$	are the mean and pulsation components of the output voltage of thermoanemometer;
$k_u, k_T$	are the coefficients of sensitivity to velocity and temperature pulsations;
$a_i = k_{Ti}/k_{ui}$	are the ratio of coefficients of sensitivity;
$A_1(T_w), B_1(T_w), n = 0.4-0.5$	are the empirical coefficients;
$m = \langle T_g'^2 \rangle^{1/2} \langle u \rangle / \langle T_g - T_\infty \rangle \langle u'^2 \rangle^{1/2}$ ,	are the ratio of intensities of velocity and temperature pulsations;
$R = \langle u'T_g' \rangle / \sqrt{\langle u'^2 \rangle} \sqrt{\langle T_g' \rangle}$	are the correlation coefficient;
$\epsilon_{k_u}, \epsilon_{k_T}, \epsilon_e, \epsilon_u$	are the relative rms errors for $k_u, k_T, \sqrt{\langle e'^2 \rangle}$ , and $\sqrt{\langle u'^2 \rangle}$ .

#### Indices

g	is the parameters of gas;
w	is the filament;
$\infty$	is surrounding medium.

#### LITERATURE CITED

1. G. S. Antonova, L. A. Vulis, and P. V. Chebyshev, *Tekh. Otchet Tsentr. Aero-Gidrodin. Inst.*, No. 34 (1954).
2. A. L. Genkin, V. I. Kukes, and L. P. Yarin, *Teplofiz. Vys. Temp.*, **14**, No. 1 (1976).
3. G. E. Hudson, *Statistics for Physicists* [Russian translation], Mir, Moscow (1976).
4. DISA Instruction and Service Manual for Type 55D01 Anemometer Unit, DISA, Denmark (1968).
5. V. I. Kukes and L. P. Yarin, *Heat and Mass Exchange V. Materials of Fifth All-Union Conference on Heat and Mass Exchange, Minsk, May 1976* [in Russian], Vol. 1, Part II, Minsk (1976), p. 167.